Module-1

Vector and tensor analysis

Q1: How many components are present in a vector quantity?

A1: Three

Q2: How many components are present in a 3rd order tensor quantity?

A2: 27

Q3: Determine free indices and dummy indices in following expressions.

a. $u_j \delta_j . v_i \delta_i + v_k$

Ans: *i* and *j* are the dummy indices and *k* is the free index.

b. $\tau_{ij} \delta_i \delta_j + v_k \delta_k$

Ans: *i*, *j* and *k* are the dummy indices

c. $\in_{ijk} \delta_k$

Ans: *i* and *j* are the free indices and *k* is the dummy index.

d. $\in_{ijk} v_j w_k \delta_i$

Ans: *i*, *j* and *k* all are dummy indices.

Q4: What is the order of tensor of Kronecker delta?

A4: Zero order

Q5: What is the order of tensor of alternating unit tensor?

A5: First order

Q6: What is the order of tensors if we add or subtract two tensor quantities?

A6: We can only add or subtract same order of tensor quantities and order of resultant tensor is also same.

Q7: What is the order of tensor and direction for the following operations?

a. Cross product of two vectors

Ans: first order and normal to the plane where both vector lies.

b. Dot product of two 2nd order tensors.

Ans: 2nd order tensor quantity, first direction is shown by first indices of first tensor and second direction is shown by second indices of second tensor.

c. Dot product of two vectors.

Ans: Zero order tensor.

Q8: If 1, 2, and 3 represent coordinates of a three dimensional cartesian coordinate system then solve following expressions.

b. $\delta_1 . \delta_2$

Ans: $\delta_{12} = 0$

d. $\delta_1 . \delta_1$

Ans: $\delta_{11} = 1$

f. $\delta_1 \times \delta_1$

Ans: $\in_{123} \ \delta_3 = \ \delta_3$

h. $\delta_1 \times \delta_3$

Ans: $\in_{132} \ \delta_2 = -\delta_2$

j. $\delta_3 \times \delta_2$

Ans: $\in_{321} \ \delta_1 = -\ \delta_1$

l. $\delta_1 \delta_2$

Ans: $\delta_1 \delta_2$

n. $\delta_1 \times \delta_1$

Ans: $\in_{112} \ \delta_2 = 0$

Q9: What is the significance of curl of a vector field?

A9: The curl of a vector field represents infinitesimal rotation of 3 dimensional vector fields.

Q10: What is the significance of divergence of a vector operator?

A10: The physical significance of the divergence of a vector field is the rate at which "density" of that field exist at given region of space.

Q11: What is the meaning of Laplacian operator?

A11: Laplacian operator is a differential operator which gives the divergence of the gradient of a function.

Q12: What is the dyadic product of a vector and tensor?

A12 3^{rd} order tensor.

Q13: What is the divergence of a tensor field?

A13 Vector quantity
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Q14: How can you decide the coordinate system for solving transport problem?

A14: Based on the geometry of system and boundary conditions.

Q15: Give the coordinate systems for solving the following problems.

b. Flow in pipe.

Ans: Cylindrical coordinate system

d. Flow in rectangular channel.

Ans: Rectangular coordinate system

f. Heat transfer (axially) in a cone.

Ans: Spherical coordinate system

h. Coating on a wire.

Ans: Cylindrical coordinate system

j. Heat transfer in a nuclear fuel road.

Ans: Cylindrical coordinate system

l. Mass transfer in a spherical catalyst.

Ans: Spherical coordinate system.

Vector & Tensors

1) If $|\vec{u}.\vec{v}| = |\vec{u}\ X\ \vec{v}|$, then the angle between $\vec{u}$ and $\vec{v}$ will be

a) π
b) $\pi/2$
c) $\pi/4$
d) 0

2) State whether the statement is true or false. $\vec{u}.\vec{v} = \vec{v}.\vec{u}$

a) True
b) False

3) State whether the statement is true or false. $\vec{u}\ X\ \vec{v} = \vec{v}\ X\ \vec{u}$

a) True
b) False

4) State whether the statement is true or false. $\bar{\bar{\sigma}}.\vec{v} = \vec{v}\ .\bar{\bar{\sigma}}$

a) True
b) False

5) If $\vec{u} = x\ \overrightarrow{\delta_1} + y\ \overrightarrow{\delta_2} + z\ \overrightarrow{\delta_3}$, then $\nabla.\vec{u}$ is equal to

a) 0
b) 1
c) 3
d) 2

6) If $\vec{u} = x\ \overrightarrow{\delta_1} + y\ \overrightarrow{\delta_2} + z\ \overrightarrow{\delta_3}$, then $\nabla\ X\ \vec{u}$ is equal to

a) 0
b) 2
c) 1
d) 3

7) If $\vec{u} = x^2\ \overrightarrow{\delta_1} + xye^x\ \overrightarrow{\delta_2} + \sin z\ \overrightarrow{\delta_3}$, then $\nabla.(\nabla\ \mathrm{X}\ \vec{u})$ is equal to

a) $x + \cos z$
b) 0
c) e^x
d) $e^z + \cos z$

8) If $r^2 = x^2 + y^2 + z^2$, then $\nabla.(\frac{\vec{r}}{r})$ is equal to

a) 0
b) r^2
c) $3r$
d) 2/r

9) Find $\delta_{ij}\delta_{ji} =$

a) 0

b) 6
c) 3
d) 2

10) Calculate $\delta_{ij}\delta_{ji} =$

a) 6
b) 9
c) 3
d) None of the above

11) Calculate $\delta_{i1}\delta_{ij}\delta_{j1} =$

a) 1
b) 3
c) 9
d) None of the above

12) Find $\in_{ijk}\in_{ijk}=$

a) 0
b) 9
c) 3
d) 6

13) $\in_{ijk}\in_{pjk}=$

a) δ_{ip}
b) $2\delta_{ip}$
c) 0
d) None of the above

14) In the following: $A_{ijk}B_{jk}C_k$. Identify the free index/indices

a) i
b) j
c) k
d) j,k
e) None

14) In the following: $A_{ij}B_{jkl}C_k = D_{jl}E_{ijk}F_k$. Identify the dummy index/indices

a) i, l
b) j
c) k
d) j,k
e) None

Ans Key

Questions	Answers
1	c
2	a
3	b
4	b
5	c
6	a
7	b
8	d
9	c
10	b
11	a
12	d
13	b
14	a
15	c